GEOMATICS ENGINEERING DEPARTMENT

SECOND YEAR GEOMATICS

GEODESY 2 (GED209)

LECTURE NO: 5

DATUM TRANSFORMATION AND DATUM SHIFT

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Overview Of Previous Lecture

- Datums
- Types of Datums
- Horizontal Datums
- Vertical Datums
- Geoid and its Significance
- Summary

Overview Of Today's Lecture

- What is Transformation?
- Datum Transformation
- Transformation Models
- Solution of Equations of Transformation Models
- Datum Shift
- Significance of Datum Transformation
- Take Home Assignment 2

Expected Learning Outcomes

- Understanding the concept and purpose of transformation in geodesy.
- Exploring datum transformations, which are used to convert coordinates from one datum to another.
- Understanding the properties and applications of similarity transformation models in geodesy.
- Understanding the importance of datum transformation in geodetic surveys, cartography, and spatial data integration.
- Exploring the challenges and considerations involved in datum transformations, such as accuracy, distortion, and coordinate system compatibility.

What is Transformation?

What is Transformation?

- Three-dimensional (3D) conformal transformations, also known as similarity transformations (since conformal transformations preserve shape and angles between vectors in space remain unchanged) are commonly used in surveying, photogrammetry and geodesy.
- For instance, in engineering surveying applications 3D transformations are used measure objects (e.g., sections of elevated roadways) off-site before they are moved on-site to make sure they will fit with existing construction, and in tunnelling operations, 3D transformations are used to control the direction and orientation of tunnel boring machines.

What is Transformation?

- In photogrammetry they are used in the (interior and exterior) orientation of digital images of structures and aerial photographs.
- In geodesy, the main thrust of this paper, 3D transformations are used to convert coordinates related to *one geodetic datum to another*, and this operation is commonly known as ***datum transformation.***
- In such applications, the rotations between the two 3D coordinate axes are small (usually less than 1 second of arc) and certain approximations are used to simplify rotation matrices.

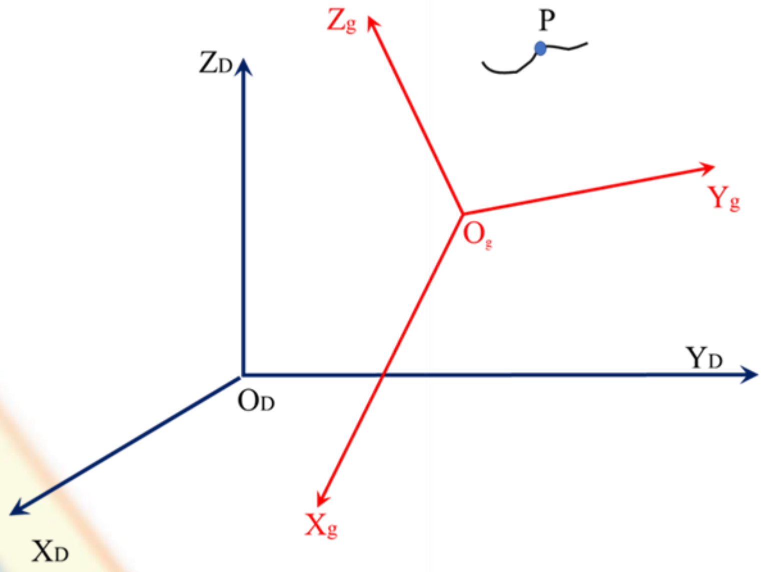
WHAT IS TRANSFORMATION?

- These simplified matrices are a common feature of the Bursa–Wolf and the Molodensky–Badekas transformations.
- The names of the two transformations are an acknowledgement to the authors M. Bursa (1962), H. Wolf (1963), M.S. Molodensky et al. (1962) and J. Badekas (1969) of technical papers and reports on transformation methods related to the orientation of reference ellipsoids and 3D geodetic networks.

WHAT IS TRANSFORMATION?

- 3D conformal transformations are often given in the form: -

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_2 = s.R_{ZYX} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_1 + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix} \quad (1)$$

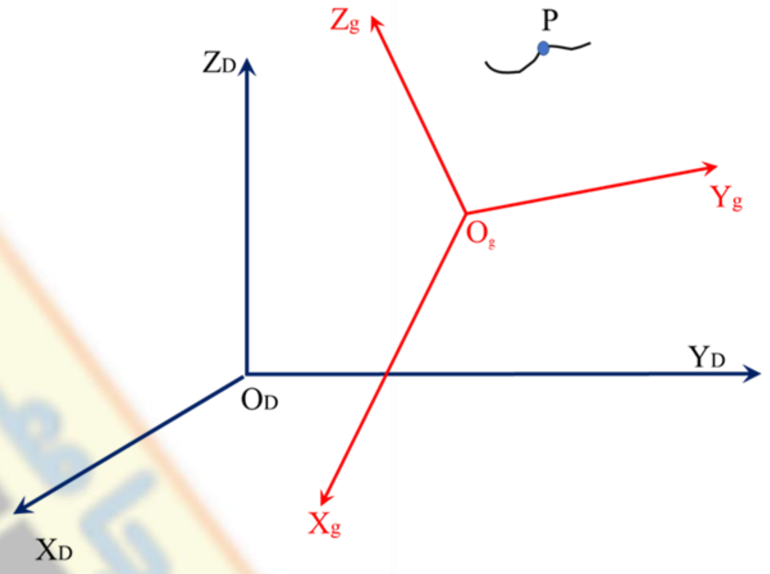


- The subscripts $[\,]_1$ and $[\,]_2$ refer to the X, Y, Z Cartesian coordinates of systems 1 and 2 respectively. s is a scale factor, R_{ZYX} is a 3×3 rotation matrix (the product of rotations r_X, r_Y, r_Z about the coordinate axes) and t_X, t_Y, t_Z are translations between the origins of the two systems measured in the directions of the system 2 coordinate axes.

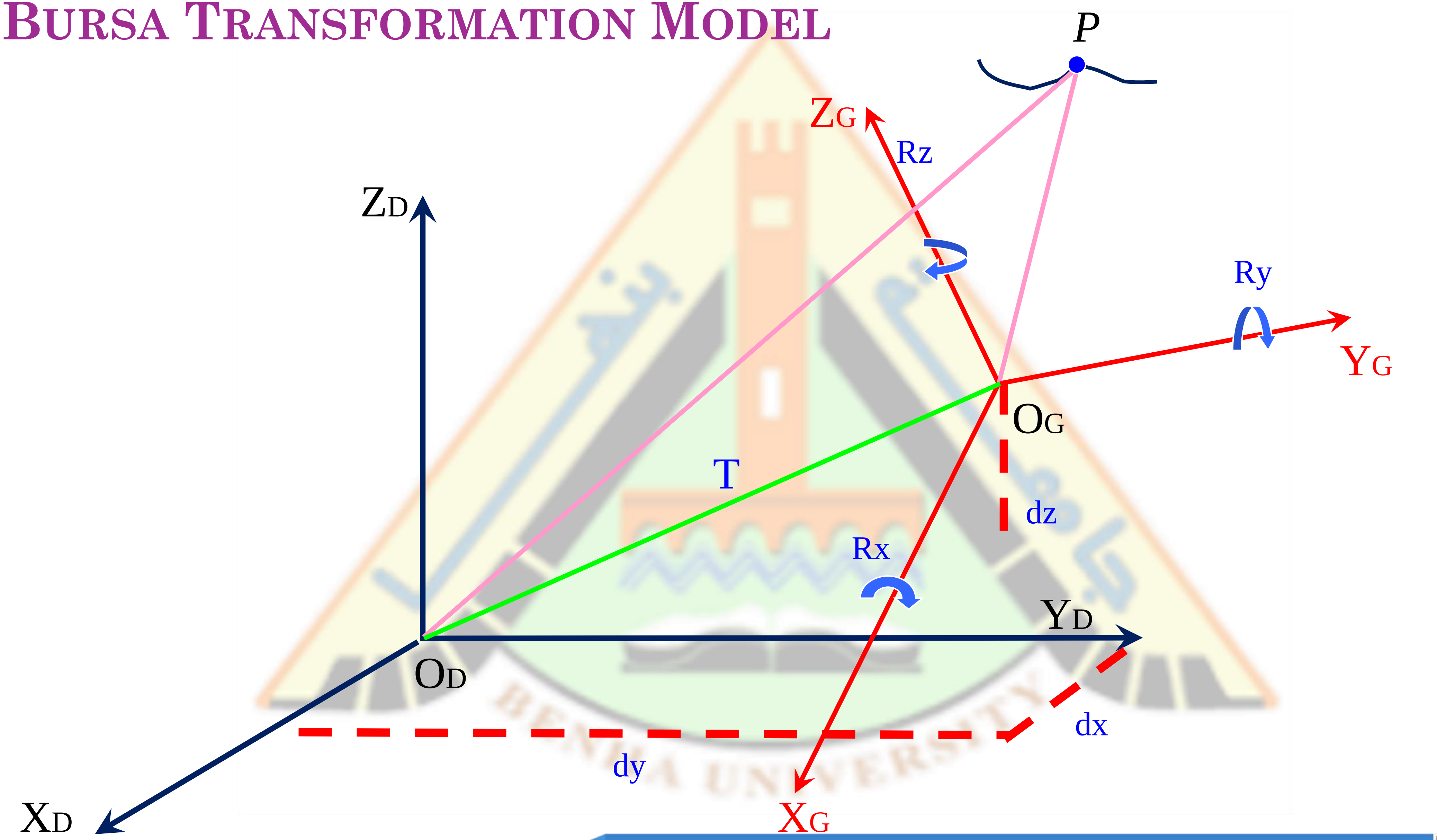
Transformation Models

MATHEMATICAL NOTATIONS

- Consider two systems: Satellite Doppler coordinate system $[XD, YD, ZD]$, and Geodetic coordinate system $[XG, YG, ZG]$
- The determined three-dimensional satellite Doppler coordinates are assumed to be the average terrestrial system. Also consider the geodetic system as the reference frame of the terrestrial network.



(1) Bursa Transformation Model



(1) Bursa Transformation Model

- $F = T + (1 + \Delta)R \times G - D = 0$
- $F = T + RG + \Delta\ G + G - D = 0$
- $R = R_z\ (B_z).R_y\ (B_y).\ R_x\ (B_x)$

Such that:

T .... Translation vector

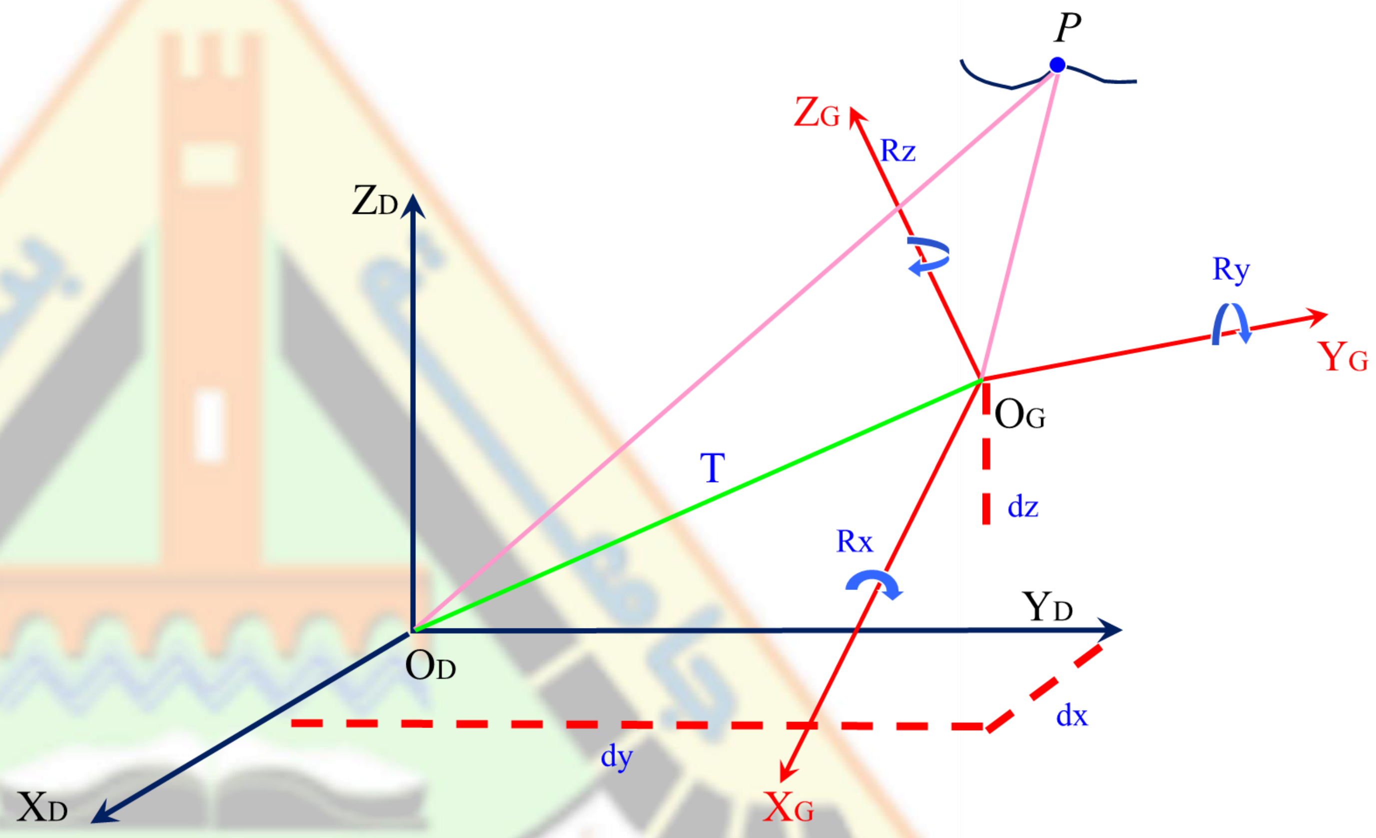
R .... Rotation matrix

Δ .... Scale factor

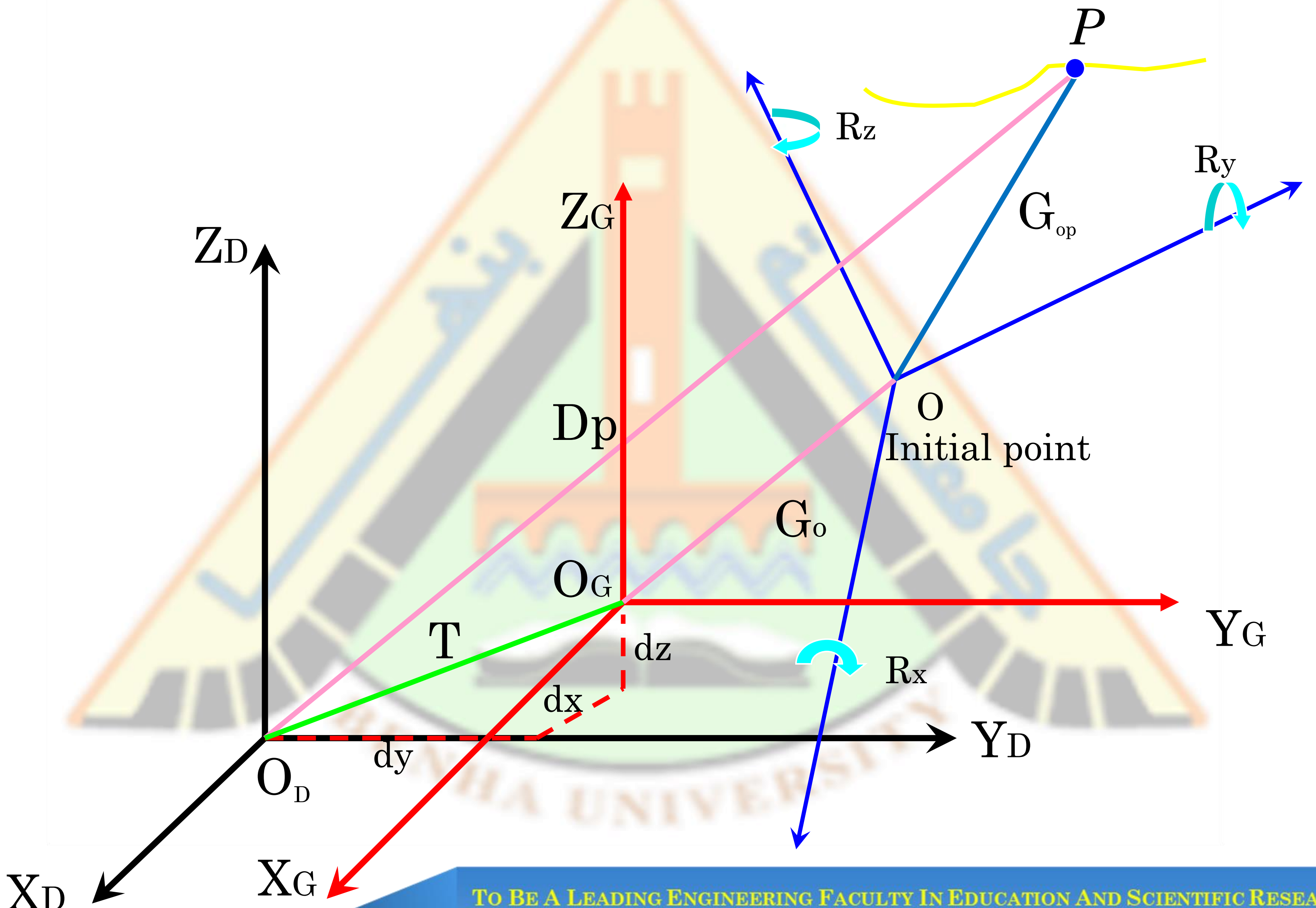
$$R_x\ (Bx) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos B_x & \sin B_x \\ 0 & -\sin B_x & \cos B_x \end{vmatrix}$$

$$R_y\ (By) = \begin{vmatrix} \cos By & 0 & -\sin B_y \\ 0 & 1 & 0 \\ \sin B_y & 0 & \cos B_y \end{vmatrix}$$

$$R_z\ (Bz) = \begin{vmatrix} \cos B_z & \sin B_z & 0 \\ -\sin B_z & \cos B_z & 0 \\ 0 & 0 & 1 \end{vmatrix}$$



(2) Molodensky–Badekas Transformation Model



(2) Molodensky–Badekas Transformation Model

$$F = T + G_o + \Delta G_{op} + QG_{op} - D_p = 0,$$

Where: -

$G_{op} = G_p - G_o$ ……….. Terrestrial position vector differences

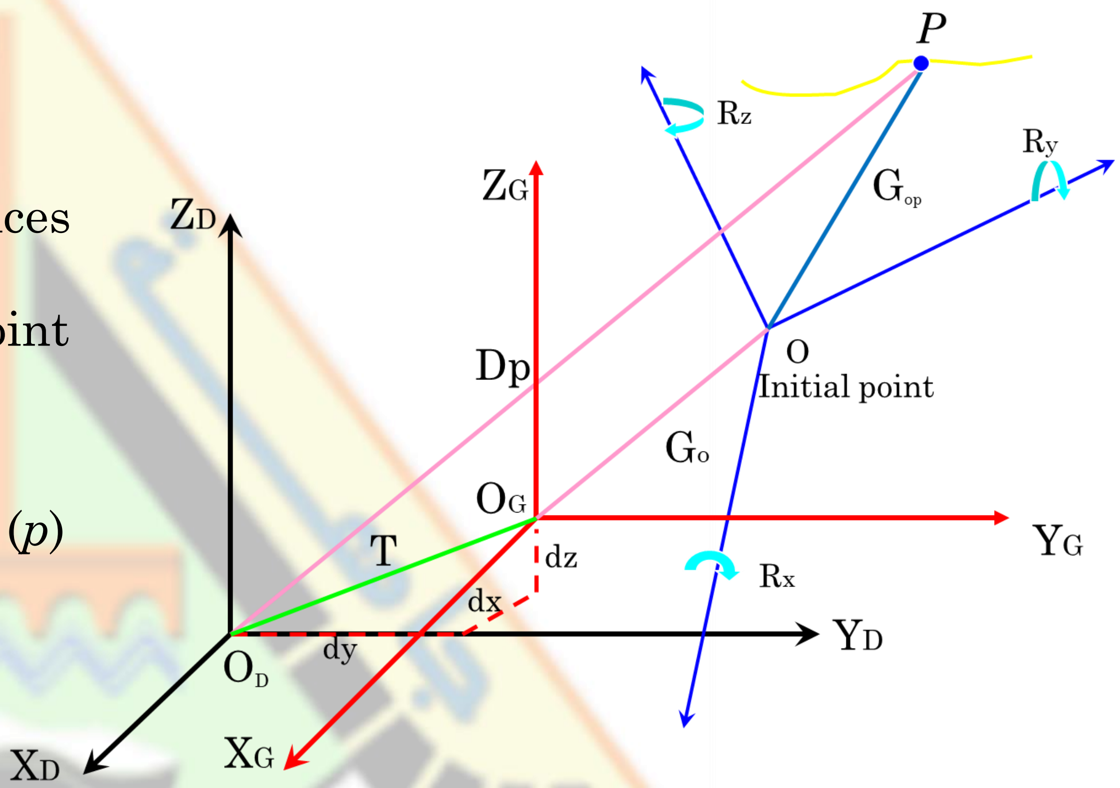
G_o ……………………. The position vector of the initial point o in the G-system

G_p ……………………. The position vector of the i^{th} point (p) from the initial point.

T ……… ……… Translation vector

Q ……… ……… Rotation matrix

Δ ……… ……… Scale factor



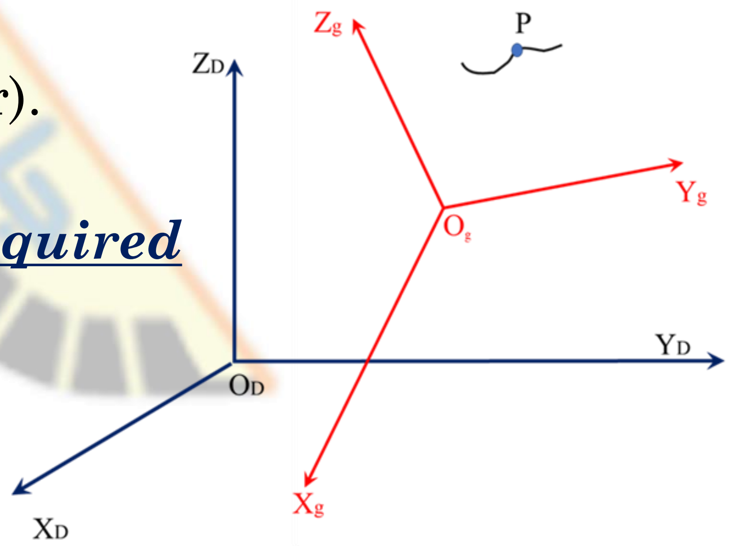
Solution of Equations of Transformation Models

What is the Number of Unknowns?

How to transform the positional coordinates of point P from $[XD, YD, ZD]$, to Geodetic coordinate system $[XG, YG, ZG]$?

- The coordinates of point P in the $[XG, YG, ZG]$ system should be: Rotated in 3D, Translated in 3D, and scaled.
- There are 7 unknowns (3 rotations, 3 translations, and scale factor).

Then, a minimum of 3 common points are required



Calculation Of Transformation Parameters

- Number of common points: 3
- Number of equations: 3 × 3 = 9
- Number of unknowns: 7
- Then, redundancy is 2 (degree of freedom)
- Least squares solution is a must.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right) \begin{bmatrix} V_{XG} \\ V_{YG} \\ V_{ZG} \\ V_{XD} \\ V_{YD} \\ V_{ZD} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & X_G & Y_G & -ZG & 0 \\ 0 & 1 & 0 & Y_G & -XG & 0 & Z_G \\ 0 & 0 & 1 & Z_G & 0 & X_G & -YG \end{bmatrix} \begin{bmatrix} d_X \\ d_Y \\ d_Z \\ \Delta \\ B_Z \\ B_Y \\ B_X \end{bmatrix} + \begin{bmatrix} X_G - XD \\ Y_G - YD \\ Z_G - ZD \end{bmatrix} = 0$$

B ***V*** ***I*** ***A*** ***X*** ***W***

Calculation Of Transformation Parameters

The solution of this system is given by:

$$\boldsymbol{X} = -(\boldsymbol{A}^T\boldsymbol{M}^{-1}\boldsymbol{A})^{-1}\ (\boldsymbol{A}^T\boldsymbol{M}^{-1}\boldsymbol{W})$$

$$\boldsymbol{V} = -\boldsymbol{P}^{-1}\boldsymbol{B}^T\boldsymbol{M}^{-1}\ (\boldsymbol{AX} + \boldsymbol{W})$$

$$\boldsymbol{M} = \boldsymbol{B}\ \boldsymbol{P}^{-1}\ \boldsymbol{B}^T$$

$$\sigma^2 = \frac{(\boldsymbol{V}^T\boldsymbol{P}\boldsymbol{V})}{DF}$$

$$\left\langle \begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right\rangle \begin{vmatrix} V_{XG} \\ V_{YG} \\ V_{ZG} \\ V_{XD} \\ V_{YD} \\ V_{ZD} \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 & X_G & Y_G & -ZG & 0 \\ 0 & 1 & 0 & Y_G & -XG & 0 & Z_G \\ 0 & 0 & 1 & Z_G & 0 & X_G & -YG \end{vmatrix} \begin{vmatrix} d_X \\ d_Y \\ d_Z \\ \Delta \\ B_Z \\ B_Y \\ B_X \end{vmatrix} + \begin{vmatrix} X_G - XD \\ Y_G - YD \\ Z_G - ZD \end{vmatrix} = 0$$

B **V** **I** **A** **X** **W**

Take Home Assignment 2

Take Home Assignment 2

- Given 5 common points between a pair of geodetic datums, compute the seven transformation parameters using Bursa and Molodeniski-Badekas models.
- Each student is required to:

1. Choose only 3 common points for calculation.
2. Write down his own code to perform transformation.
3. Submit a detailed report including code, selected points (a plot is a must), results, and analysis.

Point ID	World (WGS84)			National (Helmert1906)		
	φ°	λ°	h (m)	φ°	λ°	h (m)
P1	29.4251	32.3412	858	29.42508785	32.34117889	858.252
P2	23.2354	34.1325	654	23.23538725	34.13247889	654.401
P3	24.0056	25.5214	742	24.00558731	25.52137889	742.382
P4	30.8941	26.4812	588	30.89408801	26.48117889	588.217
P5	25.0247	29.0582	136	25.02468740	29.05817889	136.357

Take Home Assignment 2 - Submission Guidelines

- **Each student is required to: -**

a) Write down his own code to perform datum transformation.

b) Generate a 2D scatter plot using the station coordinates.

c) Submit a detailed report including code, results, and analysis.

d) *A report should start with a title page indicating student Name, Number, Section No, etc.,*

e) **Deadline**: The reports should be delivered to your tutor on or before Tuesday 26th March 2024.

The report must reflect the understanding of each student to the tutorial and copied versions will be deprecated. What you have learned in this assignment may be re-assessed in the final exam.

DATUM SHIFT

Datum Shift

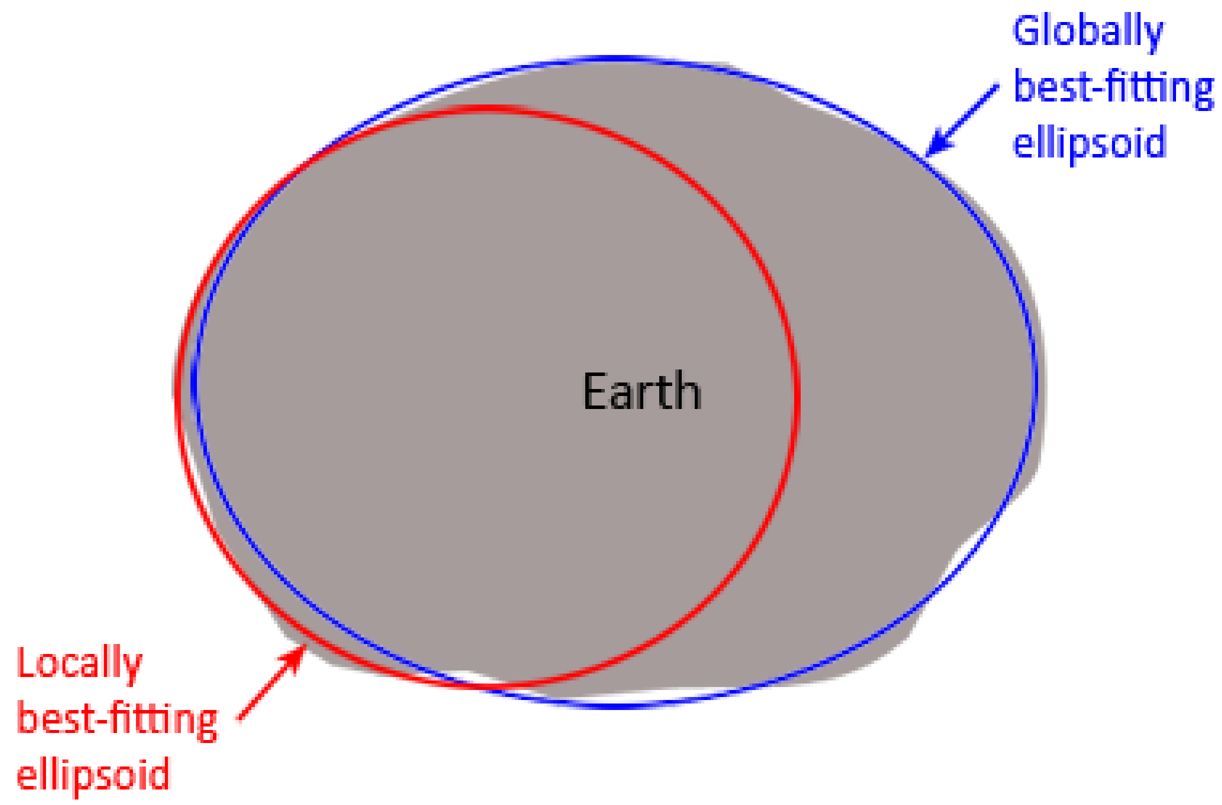
- The rectangular coordinates X, Y, Z can be obtained from the geodetic coordinates φ, λ, h for a point outside the ellipsoid:

$$X = (N + h)\cos\varphi\cos\lambda$$
$$Y = (N + h)\cos\varphi\sin\lambda$$
$$Z = (N(1 - e^2) + h)\sin\varphi$$

- It is **assumed that the center of this ellipsoid coincides with the earth center of gravity**, that is, geocentric datum.
- Suppose that we define the same dimensions (semi-major axis a, and the flattening f) for another reference ellipsoid, its center does not coincide with the earth's center of gravity, but that the axis of the ellipsoid is parallel to the earth's axis of rotation.

DATUM SHIFT

- Geodetic Datum



DATUM SHIFT

- Let the coordinates of this center w.r.t the *XYZ* system are X_o, Y_o, Z_o. Then,

$$X = Xo + (N+h)\cos\varphi\cos\lambda$$
$$Y = Yo + (N+h)\cos\varphi\sin\lambda,$$
$$Z = Zo + \left(N\left(\frac{b^2}{a^2}\right) + h\right)\sin\varphi$$

Where the principal radius of curvature in the prime vertical is known from the ellipsoid geometry:

$$N = a\,(1 - e^2\,sin^2\,\varphi)^{-0.5} = a\,(1 - f(2-f)sin^2\,\varphi)^{-0.5}$$

- Also, the following approximation will be used letter in the differential formulas

$N = a(1+ f \sin \varphi + \ldots)$, and

$N\,(1–e\,) = N\,(1–2f +f\,) = a(1–2f +f \sin \varphi +)$

which leads to

$N \approx (1–e\,)N = a$

Geodesy 2 - Dr. Eng. Reda Fekry

DATUM SHIFT

If we vary the geodetic coordinates by small amount δφ , δλ , and δ h, and if we also alter the geodetic datum, reference ellipsoid, by δa , δf , and its position by small translation, parallel displacement, δx_o , δy_o , and δz_o . Then the rectangular coordinates *X, Y, Z* change by:

$$\begin{vmatrix} \delta X \\ \delta Y \\ \delta Z \end{vmatrix} = \begin{vmatrix} \delta X_o \\ \delta Yo \\ \delta Zo \end{vmatrix} + \begin{vmatrix} \delta X/\delta\varphi & \delta X/\delta\lambda & \delta X/\delta h \\ \delta Y/\delta\varphi & \delta Y/\delta\lambda & \delta Y/\delta h \\ \delta Z/\delta\varphi & \delta Z/\delta\lambda & \delta Z/\delta h \end{vmatrix} \cdot \begin{vmatrix} \delta\varphi \\ \delta\lambda \\ \delta h \end{vmatrix} + \begin{vmatrix} \delta X/\delta a & \delta X/\delta f \\ \delta Y/\delta a & \delta Y/\delta f \\ \delta Z/\delta a & \delta Z/\delta f \end{vmatrix}$$

$$\delta X = \delta Xo + R_1 \times \delta C + R_2 \times \delta E$$

DATUM SHIFT

Considering that the position of the point in space remains unchanged, then $\delta X = 0$, the change of the geodetic coordinates φ, λ, h can also be represented as a function of the variation in the geodetic datum (a, f, x_0, y_0, z_0) as follows:

$$R^{-1}\delta Xo = R_1^{-1}R_1\delta C - R_1^{-1}R_2 \times \delta E$$

Then,

$$\delta C = -R_1^{-1}\,\delta Xo - R_1^{-1}\,R2\,\delta E$$

$$\begin{vmatrix}\delta\varphi\\ \delta\lambda\\ \delta h\end{vmatrix} = \begin{vmatrix}\frac{\sin\varphi\cos\lambda}{a} & \frac{\sin\varphi\sin\varphi}{a} & \frac{-\cos\varphi}{a}\\ \frac{\sin\lambda}{a\cos\varphi} & \frac{-\cos\lambda}{a\cos\varphi} & 0\\ -\cos\varphi\cos\lambda & -\cos\varphi\sin\lambda & -\sin\varphi\end{vmatrix}.\begin{vmatrix}\delta Xo\\ \delta Yo\\ \delta Zo\end{vmatrix} + \begin{vmatrix}0 & 2\cos\varphi sin\varphi\\ 0 & 0\\ -1 & asin^2\varphi\end{vmatrix}.\begin{vmatrix}\delta a\\ \delta f\end{vmatrix}$$

DATUM SHIFT

It is also possible to represent the change $\delta\varphi, \delta\lambda, \delta h$ of the geodetic coordinates φ, λ, h as a function of the variation $\delta\varphi_1, \delta\lambda_1, \delta h_1$ at the initial point $\varphi_1, \lambda_1, h_1$ instead of $\delta Xo, \delta Y_o, \delta Z_o$.

The translation vector δXo can be given in terms of the given $\delta\varphi_1, \delta\lambda_1, \delta h_1$ at the initial point using equation (7-19) and setting:

$$\delta X = 0, \delta C = \delta C_1, R1 = R_{1i}, R_2 = R_{2i},$$

And after rearrange it will take the form

$$\delta Xo = -R_{1i} . \delta Ci - R_{2i} . \delta E$$

Now inserting equation (7-22) in (7-21), we get:

$$\delta C = R_1^{-1} R_{1i} \delta Ci + (R_1^{-1} R_{2i} - R_1^{-1} R_2) \delta E$$

DATUM SHIFT

Where,

$$R_1^{-1}R_{1i} = \begin{vmatrix} \cos\varphi\cos\varphi_i + \sin\varphi\sin\varphi_i\cos(\lambda - \lambda_i) & -\sin\varphi\sin(\lambda-\lambda_i) & \dfrac{-[\sin\varphi\cos\varphi_i\cos(\lambda-\lambda_i) + \sin\varphi_i\cos\varphi]}{a} \\ \dfrac{\sin\varphi_i\sin(\lambda-\lambda_i)}{\cos\varphi} & \dfrac{\cos\varphi_i\cos(\lambda-\lambda_i)}{\cos\varphi} & \dfrac{-\cos\varphi_i\sin(\lambda-\lambda_i)}{a\cos\varphi} \\ a(\sin\varphi\cos\varphi_i - \cos\varphi\sin\varphi_i\cos(\lambda-\lambda_i) & a[\cos\varphi\sin(\lambda-\lambda_i)] & \sin\varphi\sin\varphi_i + \cos\varphi\cos\varphi_i\cos(\lambda-\lambda_i) \end{vmatrix}$$

And

$$R_1^{-1}\,R_{2i} \,\_\, R_1^{-1}\,R2 = \begin{vmatrix} \cos\varphi\sin\varphi_i - \sin\varphi\cos\varphi_i\cos(\lambda-\lambda_i) & -a\sin\varphi\cos\varphi_i\, sin^2\varphi_i\cos(\lambda-\lambda_i) + \cos\varphi sin^2\varphi_i - 2\cos\varphi\cos\varphi_i \\ -\cos\varphi_i\sin(\lambda-\lambda_i) & -a\cos\varphi_i\, sin^2\varphi_i\sin(\lambda-\lambda_i) \\ \sin\varphi\sin\varphi_i + \cos\varphi\cos\varphi_i\cos(\lambda-\lambda_i) & a\,[\cos\varphi\cos\varphi_i\, sin^3\varphi_i + \sin\varphi\, sin^3\varphi_i - 2\sin\varphi\sin\varphi_i] \end{vmatrix}$$

$$\delta X = \delta Xo + R1\,\delta C + R2\,\delta E$$

But $\delta X = 0$ then,

$$\delta Xo = -R_1\,\delta C - R_2\,\delta E$$
$$R_1^{-1}\,\delta Xo = R_1^{-1}\,R_1\,\delta C - R_1^{-1}\,R_2\,\delta E$$
$$\delta C = R_1^{-1}\,\delta Xo - R_1^{-1}R_2\,\delta E$$
$$\delta C = R_1^{-1}\,(-R_{1i}\,\delta Ci - R_{2i}\,\delta E) - R_1^{-1}R_2\,\delta E$$
$$\delta C = R_1^{-1}\,R_{1i}\,\delta Ci - (R_1^{-1}R_{2i} - R_1^{-1}R_2)\delta E$$

Datum Shift

The final form of this equation expresses the variations $\delta\varphi, \delta\lambda, \delta h$, at an arbitrary point in terms of the variations $\delta\varphi_i, \delta\lambda_i, \delta hi$ at the initial point, and also the changes δa and δf of the parameters of the ellipsoid. Equation (7-23) can be expressed in terms of the variations of the deflection components ξ, η, and the geoid undulation N by substituting $\delta\varphi, \delta\lambda, \delta h$, by $-\delta\xi, -\delta\eta$ and δN as follows:

$$\delta\varphi = -\delta\xi$$

$$\delta\lambda \cos\varphi = -\delta\eta$$

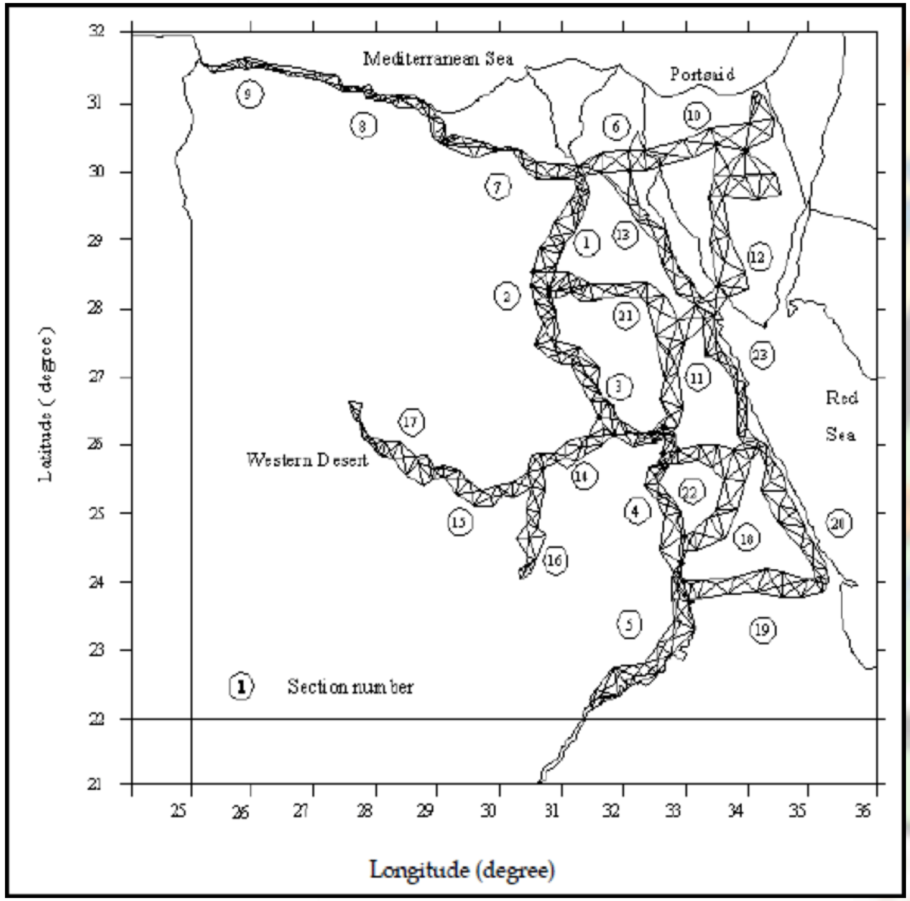
$$\delta h = -\delta N$$

This is true because the astronomical coordinates are not affected by a datum shift and remain unchanged. These formulas for the effect of a shift of the geodetic datum are the well-known Vening Meinesz transformation formula.

Significance of Datum Transformation

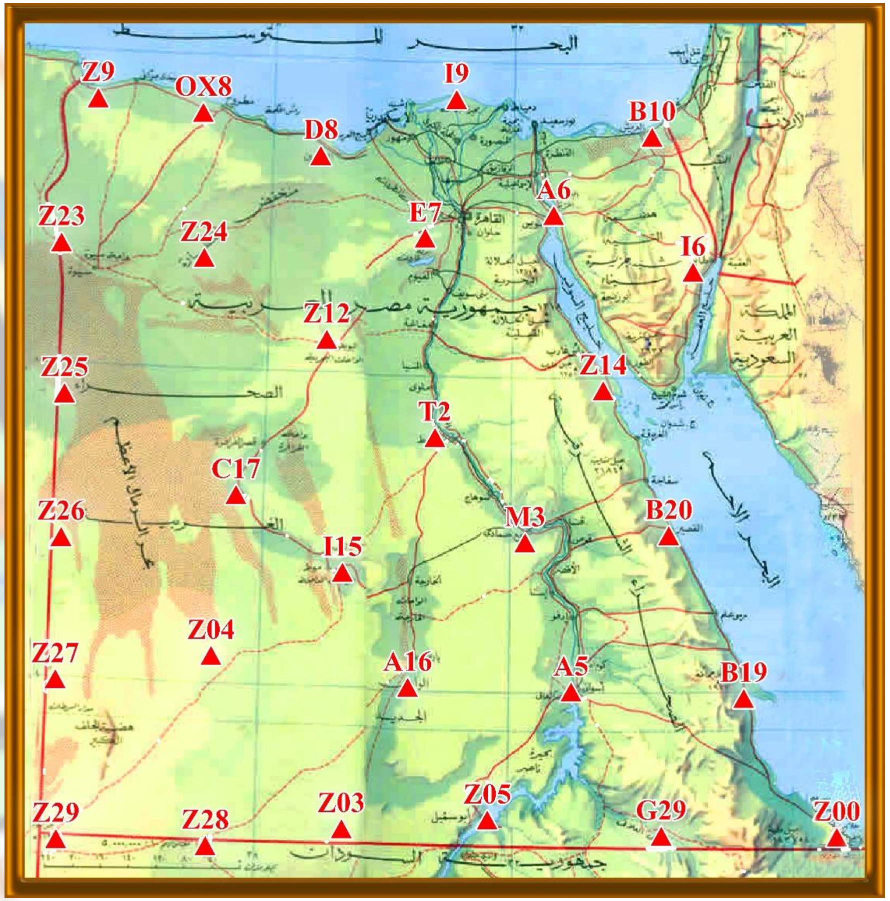
(1) Unification of Traditional and Modern Geodetic Netwrks

Traditional



First order geodetic control networks of Egypt

GNSS



HARN geodetic control networks of Egypt

Transformation Parameters Between WGS84 and Helmert 1906 In Egypt

- Used by Shell. A single transformation using position vector ***7-parameter*** geocentric transformation method with parameter values: -

- $t_X = -121.8\ m$
- $t_Y = +98.1\ m$
- $t_Z = -10.7\ m$
- $r_X = r_Y = 0\ sec$
- $r_Z = +0.554\ sec$
- $d_S = +0.2263\ ppm.$

منذ سنوات عديدة يقوم الباحثون الجيوديسيون في كل دولة بحساب قيم عناصر التحويل كلما توفرت لديهم بيانات نقاط جيوديسية معلوم إحداثياتها في كلا المرجعين (المحلي و WGS84). وتختلف دقة عناصر التحويل من دراسة لأخري طبقا لعدد النقاط المعلومة و توزيعها ودقة إحداثياتها المستخدمة في حساب عناصر التحويل ، وذلك بهدف الوصول لأدق قيم لهذه العناصر مما يسهل عملية تحويل إحداثيات الجي بي أس إلي المراجع الوطنية المستخدمة في إنتاج الخرائط لكل دولة. علي سبيل المثال توجد العديد من قيم عناصر التحويل المنشورة في جمهورية مصر العربية منهم العناصر التالية (لكلا من الدكتورة دلال النجار والدكتور جمعة داود) للتحويل من WGS84 إلي هلمرت ١٩٠٦:

$\Delta X = 125.547 \quad \pm 0.41\ m$
$\Delta Y = -113.943 \quad \pm 0.41\ m$
$\Delta Z = 10.880 \quad \pm 0.41\ m$
$Rx = -1.434 \quad \pm 0.23''$
$Ry = -1.073 \quad \pm 0.42''$
$Rz = 5.088 \quad \pm 0.43''$
$s = -5.4606 \quad \pm 1.08$ ppm (part per million)
$X_0 = 4810523.5586\ m$
$Y_0 = 2925116.9363\ m$
$Z_0 = 2962668.8097\ m$

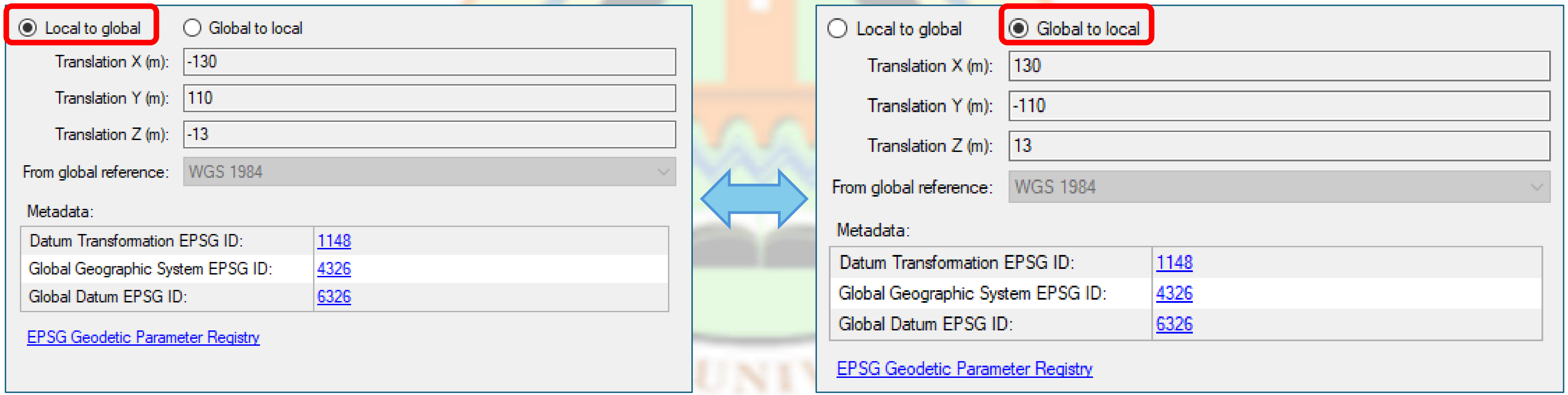
كما توجد قيم أخري نشرت حديثا للتحويل من هلمرت ١٩٠٦ إلي WGS84 - للدكاترة أحمد شاكر و عبد الله سعد و مني سعد و عمرو حنفي - وتتكون من:

$\Delta X = -88.832 \pm 0.02\ m$
$\Delta Y = 186.714 \pm 0.03\ m$
$\Delta Z = 151.82 \pm 0.01\ m$
$Rx = -1.305 \pm 2.21''$
$Ry = 11.216 \pm 1.57''$
$Rz = -6.413 \pm 1.84''$
$s = -6.413 \pm 1.84$ ppm (part per million)

Transformation Parameters Between WGS84 and Helmert 1906 In Egypt

- A single transformation using position vector ***3-parameter*** (Molodensky) geocentric transformation method with parameter values: -

- $t_X = 130\ m$
- $t_Y = -110\ m$
- $t_Z = 13\ m$



END OF PRESENTATION

THANK YOU FOR ATTENTION!